

**SIR VEN**  
(*GURU ADIWIRA KEBANGSAAN 2019*)

**SPM 2023**

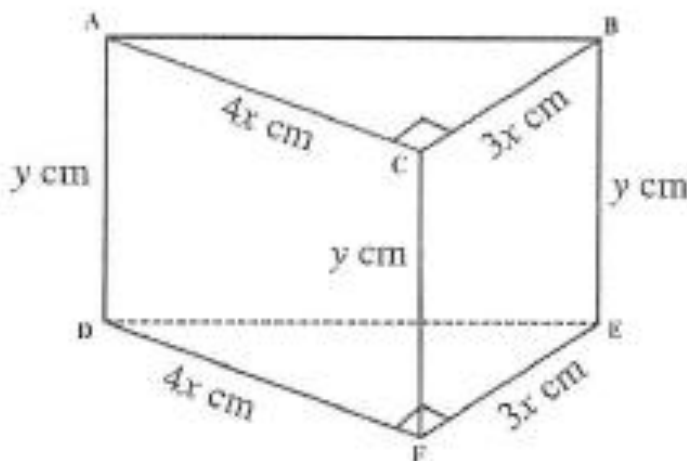
# **SOALAN RAMALAN**

## ***MATEMATIK TAMBAHAN***

### ***KERTAS 2***

**PEMBEZAAN**  
***DIFFERENTIATION***





Rajah 3  
Diagram 3

Rajah 3 menunjukkan sebuah prisma pepejal dengan keratan rentas segi tiga berserenjang dengan sisi selari  $AD$ ,  $BE$  dan  $CF$ . Panjang setiap sisi  $AD$ ,  $BE$  dan  $CF$  ialah  $y$  cm. Segi tiga mempunyai sudut tegak di  $C$  dan  $F$ . Sisi  $AC$  dan  $DF$  masing-masing mempunyai panjang  $4x$  cm. Sisi  $BC$  dan  $EF$  masing-masing mempunyai panjang  $3x$  cm. Diberi isipadu prisma itu ialah  $1500 \text{ cm}^3$ .

*Diagram 3 shows a solid prism with triangular cross section perpendicular to the parallel edges  $AD$ ,  $BE$  and  $CF$ . The length of each of edges  $AD$ ,  $BE$  and  $CF$  is  $y$  cm. the triangles have right angles at  $C$  and  $F$ . The edges  $AC$  and  $DF$  are each of length  $4x$  cm and the edges  $BC$  and  $EF$  are each of length  $3x$  cm. Given the volume of the prism is  $1500 \text{ cm}^3$ .*

- (a) Tunjukkan bahawa jumlah luas permukaan,  $S \text{ cm}^2$ , diberi oleh  
*Show that the total surface area,  $S \text{ cm}^2$ , is given by*

$$S = 12x^2 + \frac{3000}{x}$$

[ 4 markah ]

[ 4 marks ]

- (b) Cari nilai pegun bagi  $S$ .  
*The stationary value of  $S$ .*

[ 4 markah ]

[ 4 marks ]



Diberi persamaan suatu lengkung ialah  $y = x^4 - 4x^3 + 1$ .

*Given equation of a curve is  $y = x^4 - 4x^3 + 1$ .*

(a) Cari titik-titik pegun bagi garis lengkung itu. [3 markah]

*Find the stationary points of the curve.* [3 marks]

(b) Seterusnya, tentukan sifat setiap titik pegun tersebut dengan menggunakan kaedah lakaran tangen. [6 markah]

*Hence, determine the nature of each stationary points by using the tangent sketching method.* [6 marks]



(a) Diberi  $\frac{d}{dx}(3x^2 - x) = f(x)$ , cari nilai  $\int_0^1 f(x) dx$ . [3 markah]

Given  $\frac{d}{dx}(3x^2 - x) = f(x)$ , find the value of  $\int_0^1 f(x) dx$ . [3 marks]

(b) Fungsi kecerunan suatu lengkung adalah  $3x - h$ . Tangen kepada lengkung itu pada titik  $(5, 7.5)$  memotong paksi-x pada  $x = 3.75$ .  
Cari persamaan lengkung itu. [3 markah]

*The gradient function of a curve is  $3x - h$ . Tangent to the curve at point  $(5, 7.5)$  cuts the  $x$ -axis at  $x = 3.75$ .  
Find the equation of the curve.* [3 marks]



- (a) Diberi bahawa persamaan suatu lengkung ialah  $y = x^3 - 12x$ . Satu garis lurus menyentuh lengkung itu pada titik  $K(3, -9)$ . Cari persamaan garis lurus itu.

*It is given that the equation of a curve is  $y = x^3 - 12x$ . A straight line touches the curve at point  $K(3, -9)$ . Find the equation of the straight line.*

[3 markah]

[3 marks]

- (b) Diberi bahawa  $x + 2y = 4$ . Cari nilai minimum bagi  $P$  jika  $P = x^2 + xy - y^2$ .

*It is given that  $x + 2y = 4$ . Find the minimum value of  $P$  if  $P = x^2 + xy - y^2$ .*

[4 markah]

[4 marks]



Lengkung  $y = x^3 - 3x^2 - 9x + 5$  melalui titik  $A(0,5)$  dan mempunyai dua titik pusingan  $P(3,-2)$  dan  $Q$ .

*The curve  $y = x^3 - 3x^2 - 9x + 5$  passes through point  $A(0,5)$  and has two turning points  $P(3,-2)$  and  $Q$ .*

Cari

Find

- (a) kecerunan lengkung itu pada titik  $A$ . [2 markah]  
*the gradient of the curve at point  $A$ .* [2 marks]
- (b) koordinat titik  $Q$  dan tentukan sama ada  $Q$  adalah titik maksimum atau titik minimum. [5 markah]  
*the coordinates of point  $Q$  and determine whether  $Q$  is the maximum point or the minimum point.* [5 marks]
- (c) peratus perubahan dalam  $y$  jika terdapat perubahan kecil dalam  $x$  sebanyak 3% apabila  $x=5$ . [3 markah]  
*the percentage change in  $y$  if there is a small change in  $x$  by 3% when  $x=5$ .* [3 marks]



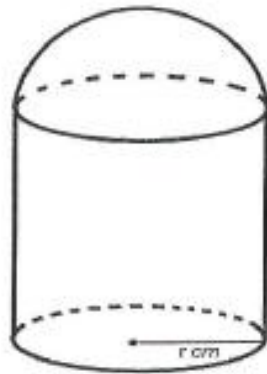
- (a) Cari persamaan tangen kepada lengkung  $y = 3x^3 - \frac{1}{x}$  pada titik  $(-1, 4)$ .

[3 markah]

Find the equation of tangent to the curve  $y = 3x^3 - \frac{1}{x}$  at the point  $(-1, 4)$ . [3 marks]

- (b) Rajah 5 menunjukkan sebuah bongkah yang terdiri daripada satu silinder, dengan jejari  $r$  cm dan satu hemisfera di atasnya.

Diagram 5 shows a solid that consists of a cylinder, of the radius  $r$  cm, surmounted by a hemisphere.



Rajah 5  
Diagram 5

Diberi isipadu silinder itu ialah  $24\pi$  cm<sup>3</sup>

Given that the volume of the cylinder is  $24\pi$  cm<sup>3</sup>.

[Luas permukaan sfera / Surface area of sphere =  $4\pi r^2$ ]

- (i) Tunjukkan bahawa jumlah luas permukaan bongkah itu,  $A$  cm<sup>2</sup>, diberi oleh

$$A = 3\pi r^2 + \frac{48\pi}{r}$$

Show that the total surface area of the solid,  $A$  cm<sup>2</sup>, is given by  $A = 3\pi r^2 + \frac{48\pi}{r}$

- (ii) Cari nilai minimum bagi jumlah luas permukaan bongkah itu.

Find the minimum value of the total surface area of the solid.

- (iii) Cari perubahan kecil bagi  $A$ , dalam sebutan  $\pi$ , jika  $r$  berubah daripada 4 kepada 4.01.

Find the small changes of  $A$ , in terms of  $\pi$ , if  $r$  changes from 4 to 4.01.

[7 markah]

[7 marks]



- (a) Diberi  $y = x(3-x)$ , cari nilai  $x$  yang memuaskan persamaan  $y \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + 12 = 0$

[4 markah]

Given that  $y = x(3-x)$ , find the value of  $x$  that satisfy the equation

$$y \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + 12 = 0 .$$

[4 marks]

- (b) Rajah 3 menunjukkan sebuah kelalang dasar bulat dengan jejari 8 cm. Tinggi paras air di dalam kelalang itu ialah  $h$  cm.

Diagram 3 shows a round-bottom flask with a radius of 8 cm. The height of the water level in the flask is  $h$  cm.



Rajah 3

Diagram 3

- (i) Tunjukkan bahawa luas permukaan membulat air,  $A$  cm<sup>2</sup>, diberi oleh  $A = \pi(16h - h^2)$ .

Show that the area of the circular water surface,  $A$  cm<sup>2</sup>, is given  $A = \pi(16h - h^2)$

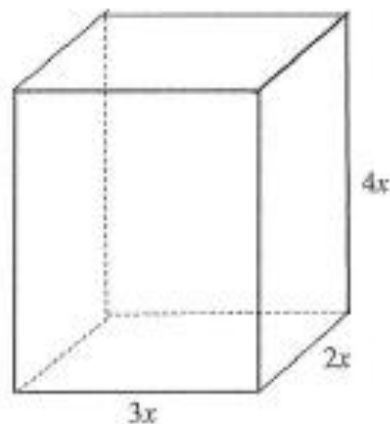
- (ii) Kelalang itu didapati bocor dan paras air menyusut pada kadar 0.5 cms<sup>-1</sup>. Hitung kadar perubahan luas permukaan membulat air itu apabila  $h = 5$ . Beri jawapan anda dalam sebutan  $\pi$ .

The flask was found leaking and the water level was decreasing at a rate of 0.5 cms<sup>-1</sup>. Calculate the rate of change of the area of circular water surface when  $h = 5$ . Give your answer in terms of  $\pi$ .



- (a) Rajah 6 menunjukkan sebuah kuboid tertutup dengan ukuran panjang  $3x$  cm, lebar  $2x$  cm dan tinggi  $4x$  cm.

*Diagram 6 shows a closed cuboid with length  $3x$  cm, width  $2x$  cm and height  $4x$  cm.*



Rajah 6

*Diagram 6*

Jika  $x$  bertambah dengan kadar  $0.1\text{cm s}^{-1}$ , cari kadar perubahan luas permukaan bila isipadu kuboid adalah  $8\text{cm}^3$ . [5 markah]

*If  $x$  increases at the rate of  $0.1\text{cm s}^{-1}$ , find the rate of change of the surface area when the volume of the cuboid is  $8\text{cm}^3$ . [5 marks]*

- (b) Diberi  $y = \frac{9}{x^2}$ , cari nilai  $\frac{dy}{dx}$  apabila  $x = 2$ . Seterusnya, anggarkan nilai hampir

bagi  $\frac{9}{(2.01)^2}$ . [5 markah]

*Given  $y = \frac{9}{x^2}$ , find the value of  $\frac{dy}{dx}$  when  $x = 2$ . Hence, estimate the approximate value of*

$\frac{9}{(2.01)^2}$ . [5 marks]

**PEMBEZAAN  
JAWAPAN**



**YIK**

$$\frac{1}{2} \times 4x \times 3x \times y = 1500$$

$\left(\frac{1}{2} \times 4x \times 3x\right)$  dan  $(3x \times y)$  dan  $(4x \times y)$  dan  $(5x \times y)$

$$S = 12x^2 + 12x \left(\frac{750}{3x^2}\right)$$

$$S = 12x^2 + \frac{3000}{x}, \text{ tertunjuk}$$

$$24x - \frac{3000}{x^2} = 0 \text{ dan selesaikan } x$$

$$x = 5$$

$$S = 12(5)^2 + \frac{3000}{5}$$

$$S = 900$$

**SABK TRIAL**

$$y = x^4 - 4x^3 + 1$$

$$\frac{dy}{dx} = 4x^3 - 12x^2$$

$$\frac{dy}{dx} = 4x^2(x-3)$$

$$\frac{dy}{dx} = 0$$

Untuk titik pegun / For stationary points

$$\frac{dy}{dx} = 0$$

$$4x^2(x-3) = 0$$

$$x = 0 \text{ atau } x = 3$$

Apabila / when

$$x = 0, \quad y = 0^4 - 4(0)^3 + 1 = 1$$

$$x = 3, \quad y = 3^4 - 4(3)^3 + 1 = -26$$

Maka, titik pegun ialah / The stationary points are

$(0, 1)$  dan  $(3, -26)$

$$\frac{d^2y}{dx^2} = 12x^2 - 24x$$

$$\text{Apabila } x = 0, \quad \frac{d^2y}{dx^2} = 12(0)^2 - 24(0) = 0$$

x	-0.1	0	0.1
$\frac{dy}{dx}$	-0.124	0	-0.116
tanda bagi $\frac{dy}{dx}$	-	0	-
Lakaran tangen			
Lakaran graf			

Maka, titik  $(0, 1)$  ialah titik lengkok balas.

$$\text{Apabila } x = 3, \quad \frac{d^2y}{dx^2} = 12(3)^2 - 24(3) = 36 > 0$$

Maka,  $(3, -26)$  ialah titik minimum.

**MRSM**

3  
(a)

$$[3x^2 - x]_0^1$$

$$(3(1)^2 - (1)) - (3(0)^2 - (0))$$

2

(b)

$$3(5) - h = \frac{7.5 - 0}{5 - 3.75}$$

$$7.5 = \frac{3}{2}(5)^2 - 9(5) + c$$

$$y = \frac{3}{2}x^2 - 9x + 15$$

**SBP**

$$\frac{dy}{dx} = 3x^2 - 12 \quad m = 15 \quad \text{dan ganti } x = 2$$

$$\frac{dy}{dx} = 3(2)^2 - 12$$

$$y + 9 = 15(x - 3)$$

$$y = 15x - 54$$

$$P = y^2 - 12y + 16 \text{ atau setara}$$

$$\frac{dP}{dy} = 2y - 12$$

$$P = (6)^2 - 12(6) + 16$$

$$-20$$

**PEMBEZAAN  
JAWAPAN**



**SABK S2**

$$\frac{dy}{dx} = 3x^2 - 6x - 9$$

$$3(0)^2 - 6(0) - 9 = -9$$

$$\frac{dy}{dx} = 3x^2 - 6x - 9 = 0$$

$$3(x-3)(x+1) = 0$$

$$x = 3 \quad x = -1$$

$$y = (-1)^3 - 3(-1)^2 - 9(-1) + 5 = 10$$

Q(-1,10)

$$\frac{d^2y}{dx^2} = 6x - 6$$

$$6(-1) - 6 = -12 < 0$$

Maka Q adalah titik maksimum.

$$y = 5^3 - 3(5)^2 - 9(5) + 5 = 10$$

$$\frac{dy}{dx} = 3x^2 - 6x - 9$$

$$\delta y \approx \frac{dy}{dx} \times \delta x$$

$$= (3(5)^2 - 6(5) - 9) \times \left(\frac{3}{100} \times 5\right)$$

$$= 5.4$$

$$\frac{\delta y}{y} \times 100 = \frac{5.4}{10} \times 100$$

$$= 54$$

**MIMS S1**

$$\frac{dy}{dx} = 9x^2 + \frac{1}{x^2}$$

$$\frac{dy}{dx} = 10$$

$$y - 4 = 10(x+1)$$

$$y = 10x + 14$$

$$\pi r^2 h = 24\pi$$

$$h = \frac{24}{r^2}$$

$$A = \pi r^2 + 2\pi r h + 2\pi r^2$$

$$A = 3\pi r^2 + 2\pi r \left(\frac{24}{r^2}\right)$$

$$A = 3\pi r^2 + \left(\frac{48\pi}{r}\right)$$

$$\frac{dA}{dr} = 6\pi r - \frac{48\pi}{r^2}$$

$$6\pi r - \frac{48\pi}{r^2} = 0$$

$$r = 2$$

$$A_{\min} = 3\pi(2)^2 + \frac{48\pi}{2}$$

$$A_{\min} = 36\pi \text{ cm}^2$$

$$\delta A = \left[ 6\pi(4) - \frac{48\pi}{(4)^2} \right] \times 0.01$$

$$= 0.21\pi$$

**MIMS S2**

$$y = x(3-x)$$

$$= 3x - x^2$$

$$\frac{dy}{dx} = 3 - 2x$$

$$\frac{d^2y}{dx^2} = -2$$

$$(3x - x^2)(-2) + 3x - 2x^2 + 12 = 0$$

$$-3x + 12 = 0$$

$$-3x = -12$$

$$x = 4$$

Katakan jejari permukaan membulat air ialah r cm.

$$r^2 = 8^2 - (8-h)^2$$

$$r^2 = 64 - (64 - 16h + h^2)$$

$$r^2 = 16h - h^2$$

Luas permukaan membulat air

$$A = \pi r^2$$

$$A = \pi(16h - h^2)$$

Tertunjuk

$$\frac{dA}{dh} = \pi(16 - 2h)$$

Dengan menggunakan petua rantai,

$$\frac{dA}{dt} = \frac{dA}{dh} \times \frac{dh}{dt}$$

$$= \pi(16 - 2h) \times -0.5$$

$$= \pi[16 - 2(5)] \times -0.5$$

$$= -3\pi \text{ cm}^2 \text{ s}^{-1}$$

**MIMS S3**

$$24x^2 = 81$$

$$x = \frac{3}{2}$$

Luas permukaan =  $2(3x)(2x) + 2(2x)(4x) + 2(3x)(4x) @ 52x^2$

$$\frac{dA}{dx} = 104x, \quad x = \frac{3}{2}$$

$$\frac{dA}{dx} = 104 \left(\frac{3}{2}\right)$$

$$\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$$

$$\frac{dA}{dt} = 104 \left(\frac{3}{2}\right) \times 0.1$$

$$15.6$$

$$\frac{dy}{dx} = -18x^{-3}$$

$$\frac{dy}{dx} = -18(2)^{-3}$$

$$\frac{dy}{dx} = -2.25$$

$$\delta y = (-2.25)(0.01)$$

$$\delta y = -0.0225$$

Nilai hampir =  $y + \frac{dy}{dx} \delta x$

$$= 2.25 + (-0.0225)$$

$$= 2.2275$$